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Analysis of Student Learning in Differential Equations Learning Area Based on Various Variables: The Case of Turkey and Ghana

Çeşitli Değişkenlere Göre Farklı Denklemler Öğrenimi Alanında Öğrenci Öğreniminin Analizi: Türkiye ve Gana Örneği

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**Çeşitli Değişkenlere Göre Farklı Denklemler Öğrenimi Alanında Öğrenci
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Abstract: Development of mathematical models for problems experienced in study areas is important for improvement in almost all scientific fields. The solution of a problem requires development of a mathematical equation (or a mathematical model) based on the properties of the problem in certain disciplines. Such an equation is generally a function with one unknown and an equation that includes the derivative (or differential) of this function. The equation which contains a function and its finite derivatives, is called a differential equation (Pisces 2018). The present study aimed to analyze the learning of college students in differential functions based on various variables in Turkey and that of Ghana. The study was conducted in two faculties of a public university (Engineering and Education Faculties) for a year in differential equations course on 300 students' exams and applications for Turkey. The achievements of those students were analyzed based on the problems experienced by the students, the conceptual comprehension and operational comprehension, reasoning skills, misconceptions, transition between representations, and basic mathematics knowledge variables. A similar study was carried out in a faculty of technical education in Ghana with 300 students' examination and application and their results were analyzed as it was done in the case of Turkey. The present study for the two countries adopted a descriptive approach based on scre-

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"COPE-Dergi Editörleri İçin Davranış Kuralları ve En İyi Uygulama İlkeleri" beyanları: Bu çalışma için herhangi bir çıkar çatışması bildirilmemiştir. Bu çalışma için etik kurul onayı gerekmemektedir. Sorumlu Yazar: Cemil İNAN

ening models. Since it was possible to reach the whole population in the study for the two countries, the complete census method was used. The main difficulties experienced by the students in each country were determined as follows: inability to conduct the operations or to conduct incorrect operations due to the lack of basic mathematical knowledge, student focus on operational comprehension instead of conceptual comprehension, inability or indecision of the students in establishing the correlation between the algebraic and graphic representations of the differential equation in transfers between representations, misconception that dependent variable should always be identified by y and independent variable should always be identified by x (this could be due to a teacher habit), and although $dx-2xydy-2y^2dy$ could be easily solved by linearization, students' preference to attempt to find the integral multiplier, leading to calculation errors. These difficulties were similar in within the two countries as if they were taught by same instructors. It can be argued that the achievements increased in the two groups of students when the applications that would eliminate these difficulties were conducted in the following topics, however a comprehensive study should be initiated to eliminate the lack of basic mathematical knowledge, and it would even be beneficial to study the OSYM selection system. It could be suggested that calculus as a concept should be taken serious to build the two groups of students competencies in differential equations.

Keywords: Differential equations, learning difficulties, OSYM selectivity, student achievements.

Öz: Çalışma alanlarında yaşanan problemler için matematiksel modellerin geliştirilmesi hemen hemen tüm bilim dallarının iyileştirilmesi için önemlidir. Bir problemin çözümü, belirli disiplinlerde problemin özelliklerine dayalı bir matematiksel denklemin (veya matematiksel bir modelin) geliştirilmesini gerektirir. Böyle bir denklem genellikle bir bilinmeyenli bir fonksiyon ve bu fonksiyonun türevini (veya diferansiyelini) içeren bir denklemdir. Bir fonksiyon ve onun sonlu türevlerini içeren denkleme diferansiyel denklem denir (Balık 2018). Bu çalışma, Türkiye ve Gana'daki üniversite öğrencilerinin çeşitli değişkenlere dayalı olarak diferansiyel fonksiyonlardaki öğrenmelerini analiz etmeyi amaçlamıştır. Araştırma, bir devlet üniversitesinin iki fakültesinde (Mühendislik ve Eğitim Fakülteleri) bir yıl süreyle diferansiyel denklemler dersinde 300 öğrencinin Türkiye'ye yönelik sınavları ve başvuruları üzerinde gerçekleştirilmiştir. Bu öğrencilerin başarıları, öğrencilerin yaşadıkları problemler, kavramsal anlama ve işlemsel anlama, muhakeme becerileri, kavram yanılgıları, temsiller arası geçiş ve temel matematik bilgisi değişkenlerine göre analiz edilmiştir. Benzer bir çalışma Gana'da bir teknik eğitim fakültesinde 300 öğrencinin sınav ve uygulaması ile gerçekleştirilmiş ve sonuçları Türkiye örneğinde olduğu gibi analiz edilmiştir. İki ülke için bu çalışma, tarama modellerine dayalı betimsel bir yaklaşım benimsemiştir. Çalışmada iki ülke için tüm popülasyona ulaşmak mümkün olduğundan tam sayım yöntemi kullanılmıştır. Her ülkede öğrencilerin yaşadıkları başlıca zorluklar şu şekilde belirlenmiştir: Temel matema-

tik bilgisinin eksikliğinden dolayı işlemleri yapamama veya yanlış işlemler yapma, öğrencinin kavramsal anlama yerine işlemsel kavramaya odaklanması, öğrencilerin başarısızlığı veya kararsızlığı, gösterimler arası transferlerde diferansiyel denklemin cebirsel ve grafik gösterimleri arasındaki korelasyonu kurarken, bağımlı değişkenin her zaman y ve bağımsız değişkenin her zaman x ile tanımlanması gerektiği (bu bir öğretmen alışkanlığından kaynaklanıyor olabilir) yanlışlığı ve $dx-2xydy-2y^2dy$ doğrusallaştırma ile kolayca çözülebilse de, öğrencilerin tercihlerinin integral çarpanını bulmaya çalışması hesaplama hatalarına yol açmaktadır. Bu zorluklar, sanki aynı öğretmenler tarafından öğretiliyormuş gibi iki ülkede de benzerdir. Aşağıdaki konularda bu güçlükleri giderecek uygulamalar yapıldığında iki grup öğrencide başarıların arttığı, ancak temel matematik bilgi eksikliğinin giderilmesi için kapsamlı bir çalışma başlatılmalı ve hatta bu konuda daha kapsamlı bir çalışma başlatılmalıdır. ÖSYM seçim sistemini incelemek için faydalıdır. İki grup öğrencinin diferansiyel denklemlerdeki yeterliklerini oluşturmak için bir kavram olarak matematiğin ciddiye alınması gerektiği önerilebilir.

Anahtar Kelimeler: Diferansiyel denklemler, öğrenme güçlükleri, ÖSYM seçiciliği, öğrenci başarıları.

1. Introduction

Developing mathematical models for problems encountered in study areas is important for the development in almost every scientific field. The solution of a problem in certain disciplines requires the formation of a mathematical equation (or mathematical model) based on the properties of the problem. Such an equation is generally an unknown function and an equation that includes the derivative (or differential) of this function. The equation, which includes a function and its finite derivatives, is called a differential equation (Pişkin 2018).

Every movement has certain rules in the nature. Most of these rules are differential movements that include at least one dependent and one independent variable; for instance it is known that rain is dependent on evaporation, seed is dependent on the sun and water, free fall is dependent on gravity and the air resistance, heating or cooling is dependent on the temperature of the environment around the object, the curves indicated by a parabola family are dependent on the equation constant, electric current is dependent on Kirchoff voltage equations, population change is dependent on the present population rate, etc. Based on these properties, it is possible to express a problem with a differential equation. Previous studies reported problems in the instruction and solution of differential equations (Anderson & Seaquist, 1999, p.1). The knowledge that mathematics could be used for certain basic events allows students to develop positive attitudes towards mathematics (Pişkin 2018b). The differential equations that provide a model for the comprehension and solution of various problems experienced in daily life could be solved with three approaches: algebraic, numerical,

and graphical (qualitative). However, previous studies reported that the instruction of this topic was limited to the algebraic approach (Arslan 2004; Artigue & Rogalski 1990; Habre 2000; Kalleher 1999; Ramirez 1995; Salam 2004).

The solution of the differential equations is usually based on the use of the formula of the general algebraic solution of equations of different types. The interpretation of the mathematical meaning of the problems is usually ignored (Boyce, 1994, p. 364). Algebraic solution of all differential equations is not possible. Thus, the instruction is limited to certain selected topic and other topics are neglected.

However, non-linear and differential equations without an analytical solution and approximate solutions or analytical interpretations of these equations are required. Today, it was reported that the same behavior continues (Arslan, 2005; Saglam, 2004). However, the differential equations introduced to mathematics in the 17th century have been solved with three different approaches. Although the algebraic solution was prominent for over a century, numerical solutions were discovered in the 18th century, followed by qualitative or graphical solutions in the 19th century (Arslan, 2005). A brief review of these methods would indicate that the general solution of the given differential equation can be obtained with an algebraic expression in the analytical approach. The general solution of a differential equation includes arbitrary constants equal to the order of the equation. Numerical solution aims to find approximate values for a particular solution of a differential equation that passes through a given point, and thus allows the unknown function to be calculated for specific values of the independent variable. In other words, in this approach, to approximate the solution of differential equation $y' = f(x, y)$ at point (x_0, y_0) , various y values for various x values are found, and thus approximate values for that solution curve are calculated within an interval. In the qualitative (graphical) approach, to solve $y' = f(x, y)$ differential equation, the objective is to collect information about the general properties and the behavior of the solution curves without solving the equation algebraically. Thus, the intersections of the solution curves, the regions where solution curves ascend (descend) based on the increasing (decreasing) independent variable, the presence or the absence of the asymptote, i.e., whether the solution curves approach zero or a finite value, and also certain integral curves for a given differential equation could be determined based on the analysis (Arslan, S.2008, p: 156). The impact of these developments on mathematics education has been limited.

The study of differential equations as a measure of students understanding of this concept have not been seriously considered in Ghana and some part of Africa. Students have been taking differential equations as a course in many discipline in the colleges of the universities. However, some few studies have been explored in the light algebra in many countries in Africa. Adu et et al., (2015) explored error analysis of a

selected senior high in the central region of Ghana using modified Newman Error Hierarchical levels. They found out that the majority of the students made comprehension errors and could not obtain the right solution. Ibrahim Abubakar (2017) analyzed some errors committed by some selected senior high schools in solving simultaneous and quadratic equation and observed that these errors were equally associated with teachers and students in Kaduna state of Nigeria. This study is intended to bring the detail picture of the comparative studies of the two countries students performance in differential and would guide instructors in the development of this concepts.

1.2. The Significance of the Present Study

The most important investment in science and technology is the investments in humankind. It is a long-term and expensive investment, thus investment in human education should be meticulous and it should be continuously analyzed and the related problems should be solved, otherwise it could lead to waste of time and resources. Differential equations in any form usually based on engineering problem which solve real life problem. Ghana as a developing country require fully competent human beings in the filed physical and applied sciences in which all need the concept of differential equation. The language or logic of science and technology is mathematics, and the branch of mathematics utilized in science and technology is the applied mathematics and differential mathematics. Differential equations and their solutions play a significant role in applied mathematics problems. The topic of differential equations should be continuously reviewed to determine the difficulties and problems and to prevent the repetition of similar behaviour. Although there are several studies on the topic in the literature, there are no studies conducted on student exams, assignments and projects and this provide vital qualitative information in the understanding of this topic of the two groups. Exam sheets are official documents and a reflection of student knowledge. It is desirable to know whether an applied education is successful and to what extent it is successful. During the educational process, the early recognition of failure and unsuccessful individuals makes it easier to take measures that will lead to improvement in these two groups performances. The degree of achievement should be made known, and failures should be revealed. Otherwise, systematic errors would continue and effort at improving the learning outcomes of these two groups will not be attained. It was considered that the present study findings would contribute to the literature in the two countries and beyond.

1.1 The Aim of the Study

Unless whether a topic is learned is determined objectively and resolutely with competence, it is not possible to determine whether the topic is really learned. The correlation between independent and independent variables is the basic knowledge and reasoning required in all disciplines. There is almost no study area that is not affected

by this strong correlation between the variables. It is not possible to claim that one is successful in that area. Reasoning is the basis of science and the concept of differential equations is its reflection on applied disciplines. The present study aimed to analyze the difficulties experienced in differential equation problems based on the two groups student exam sheets, assignments and projects that possess the quality of a document and to determine general these students from the two groups mistake and to present the appropriate recommendations needed to solve these problems.

2. Method

The present study was designed as a case study. Because, case studies allow the in-depth investigation of the topic of interest. It allows the systematic analysis of correlations between the collected data and explain these correlations within the framework of causality (Cohen and Manion, 1994). Case study methodology was used in the present study since the data were collected with student exam sheets, assignments and projects conducted during the previous five years.

2.1. The Study Group

The study group included the students attending the differential equation course at education and engineering faculties of a public university in Turkey between 2014 and 2019 academic years. The average number of students who attended this course in that period was 300 (80 primary school mathematics teaching, 40 middle school mathematics teaching, 40 mechanical engineering, 70 civil engineering, 60 electric electronics engineering students). The study was conducted on the exam sheets, assignments and projects submitted by these students. In Ghana between same period similar study was conducted in the College of Technology Education formerly of University of Education Winneba, Kumasi campus now Akenten Appiah Menka University of Skills Training and Entrepreneurial Development which is a public university. targeted students in the faculty of Technical Education. The average number of students who took the engineering mathematics III was 300. Specifically, Construction Education II 180, Auto and Mechanical Engineering Education II 50, Wood Technology II 40 and Electrical/Electronic Education II 30.

2.2. Data Collection and Analysis

The exam sheets, assignments and projects submitted by the students for five years were classified based on the subject title in a tertiary institution in Turkey and Ghana respectively. The qualitative data obtained with the questions on each topic are presented with percentages, interpretations and general mistakes. The correct solution to the problem is included for comparison.

3.Findings

It was observed that the students in Turkey selected y as the dependent variable and x as the independent variable and this was considered as an excessive generalization mistake and since the students could not control this when solving the problems, they experienced problems ($n = 87$; 29%). In Ghana same problem students ($n=78$;26%) made same generalization mistake.

Question 1: Solve the following differential equation: $2x \left(xy^2 - \frac{1}{y} \right) y' = 1$. By dividing both sides of the equation by x^2 , Bernoulli-type equation $x' + \frac{1}{y} 2x = 2x^2 y^2$ with a dependent variable x and independent variable y is obtained. And by $z = x^{-1}$ transformation, it is transformed into the $z' - \frac{2}{y} z = 2y^2$ linear equation and the general solution is obtained as follows: $2xy^3 + cxy^2 = 1$.

It was observed that the students in Turkey experienced problems in determination of the order and degree of a differential equation (by ignoring the polynomial properties). In Ghana 35% of the students were able to transformed the problem correctly that could help to determine the order and degree of the differential equation.

Question 2: Find the order and degree of the differential equation $y'' = [1 + (y')^2]^{\frac{3}{2}}$. 136 students (45.3%) in Turkey responded that it was a 1st degree and 2nd order, or 2nd degree and without an order. Similarly, in Ghana 120 students (40%) observed same as in the case of Turkey. In fact, by squaring both sides, it could be observed that it was a 2nd order and 2nd degree equation.

The achievement rate in questions on determination of the linearity of a differential equation was 59.6% ($n= 179$) for the Turkish students while in Ghana under same question the achievement rate was 58% (174). This perhaps suggest that students had similar concepts of ordinary differential equation during the course period.

Question 3: Analyze $y'' + x(y')^2 = \sin x$ for linearity. They all accepted that the differential equation as linear, in fact, the degree of its derivatives should be 1 in both groups. The two groups only focused on whether the dependent variable and its derivatives were multiplied.

Those students from Turkey generally neglected outlier/unique solutions ($n = 168$; 56%) while those from Ghana ($n=172$;57.33%) equally neglected the unique solutions.

Question 4: Analyze $y'^2 = 4y$ solution. In both groups the students were not able to distinguish that $y = (x + c)^2$ was a general solution and there were further solutions that could not be obtained by assigning arbitrary values to the c parameter.

The general solution for differential equations is represented by a curve family.

They experienced problems in solutions due to lack of analytical geometry knowledge (n = 195; 65%) for students from Turkey and (186; 62%) students from Ghana respectively.

Question 5: Find the differential equation for the family of circles with a center at line $x + y = 1$ and intersect the origin. The radius r is selected as $M(a, 1-a)$ and the equation for the circle that intersects the origin $a^2 + (1-a)^2 = r^2$ is found. And the equation of the circle that intersects any $A(x, y)$ point on the circle would be $(x-a)^2 + (y-(1-a))^2 = r^2 = a^2 + (1-a)^2$. When its derivative is taken and solved for a , then $a = \frac{x+y'(y-1)}{(1-y')}$ is written in the equation for the point on the circle, then $(\frac{x+y'(y-1)}{1-y'})^2 + (1 - \frac{x+y'(y-1)}{(1-y')})^2 = 0$ differential equation is obtained.

Although the students from Turkey did not experience difficulties in differential equations that could be solved directly with integral, they made mistakes due to missing operations in equations that could be separated into variables (n = 96; 32%). In Ghana fewer number of students encountered difficulties in solving the problem directly using integral (n=60; 20%). Again, those students from Ghana made mistake as well in the missing operations in the equation that could have led to separation of variables (n=99; 33%).

Question 6: The two separate groups experienced problems in separating the equation $\frac{dy}{dx} = \frac{y^2+xy^2}{x^2y-x^2}$ into $\frac{y-1}{y^2} dy = \frac{1+x}{x^2} dx$ by bracketing the numerator by y^2 and the denominator by x^2 . However, in the case of Ghana the number of students who couldn't separate the variables not many (n=60; 20%).

Question 7: Solve $xy' + xy^2 = x$. 38% of the students from Turkey failed in simple fractioning the equation to $\int \frac{dy}{1-y^2} = \int dx$ and $\frac{1}{2} \int \frac{1}{1-y} + \frac{1}{1+y} dy = \int dx$ while 39% students from the case of Ghana also failed the same concept.

It is interesting to notes in both cases that the students were frequently confused about derivative and integral equations when solving the differential equations (n = 147; 47%) for Turkish students and (n = 150; 50%) for Ghanaian students in that order.

Question 8: The following equations could be given as examples: $\int \frac{du}{a^2-u^2} = \frac{1}{2} \ln I \frac{u+a}{u-a} I + c$ and $\int \frac{dx}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + c$.

Most students in the case of Turkish failed in hyperbolic functions (n = 118; 39.3%), however, the case of Ghanaian students (n=180; 60%) failed same question.

Question 9 : Obtain $\cosh x = \frac{e^x + e^{-x}}{2}$ equation. For this particular question the two groups appeared confused because almost all of them could not explain how it

was obtained (using e^x and e^{-x} expansions).

The two groups Turkey and Ghana respectively were successful in the solution of homogenous functions on the same degree using, $y=ux$ transformation ($n = 204$; 68%) and ($n = 210$; 70%) in problems that could be transformed into homogenous equations when $\Delta = 0$ ($n = 202$; 67.3%) and ($n = 221$; 73.6%). However when $\Delta \neq 0$ the achievement is reduced ($n = 107$; 35.6%) for Turkey and ($n = 99$; 33.6%) for Ghana in that order. It could be inferred that the aim of axis translation is to nullify the constants was not understood in two groups certain.

Question 10: Solve $y' = \frac{-x+2y+6}{2x+2y}$ differential equation. Since $\Delta = -6 \neq 0$, the lines intersect at $(h, k) = (2, -2)$ and $x = X + h$ and $y = Y + k$ variable translation is shown on the figure in the coordinate system and in $\frac{dy}{dx} = \frac{-X+2Y}{2X+2Y}$ homogenous equation, $Y=u$ X variable and $X=x-2$, $Y=y+2$ translation will lead to $\ln I \frac{2(y+2)^2}{(x-2)^4} I + 2\sqrt{2} \arctan\left(\sqrt{2} \frac{y+2}{x-2}\right) = c_1$ general solution. Most students from Turkish preferred the second solution in the solution of $P(x,y)dx + Q(x,y)dy = 0$ differential equation ($n = 260$; 86%). 1st solution: $u(x,y) = \int P(x,y)dx + h(y)$ For the case of Ghana the majority of the students ($n=240$; 80%) utilized the second (2nd) solution approach: $u(x,y) = \int P(x,y)dx + \int Q(x,y)dy - \int \left[\frac{\partial p}{\partial y} dx \right] dy$

Question 11: Solve $(x+y)dx + (x-y)dy = 0$ differential equation. Since $\frac{\partial p(x,y)}{\partial y} = \frac{\partial Q(x,y)}{\partial x} = 1$ is a complete differential equation, $u(x,y) = \int (x+y)dx + h(y) \rightarrow h(y) = -\frac{y^2}{2} + c$ and by writing it in the equation, the solution would be $\frac{x^2}{2} + xy - \frac{y^2}{2} = c_1$, however, the students from both groups could not obtain this solution. Although both give the same result, it could be stated that the both groups students tended to use the ready equations. It can be inferred that students from both groups have wrong concepts.

In $P(x,y)dx + Q(x,y)dy = 0$ non-complete differential equation in the part of Turkish students, they hesitated whether the equation was dependent on only x or only y ($n = 178$; 59.3%). However, for Ghanaian students ($n=180$; 60%) could not identify the dependent and the independent ($n=180$; 60%).

Question 12: In the solution of $(x^2 + y^2)dx + xydy=0$ differential equation, since $\frac{\partial p(x,y)}{\partial y} = 2y$ and $\frac{\partial Q(x,y)}{\partial x} = y$, it is not a complete differential equation. It is solved with $\frac{Vx}{V} = \frac{Py-Qx}{Q} = \frac{2y-y}{xy} = \frac{1}{x}$. It can be observed that $\frac{Vy}{V} = \frac{Qx-Py}{P} = \frac{-y}{x^2+y^2+x}$ is not suitable.

It was observed that achievement in part of Turkish students was very low in problems that require functions that require expansion to power series ($n = 81$, 27%) and similar pattern was observed in the Ghanaian students ($n=72$; 24%) where only

small number of students were able to solve.

Question 13: Approximate $\int e^{x^2} dx$ integral using Taylor's theorem of the mean.

$$\text{For } \int e^{x^2} dx, e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \quad 0 < x < 1 \text{ and } e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{4!} + \frac{x^{10}}{5!}$$

$$0 < \delta < x^2 \quad \int_0^1 e^{x^2} dx = \int_0^1 (1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} \dots) dx + c = 1 + \frac{1}{3} + \frac{1}{5.2!} + \frac{1}{7.3!} + \frac{1}{9.4!} + \dots + c$$

In linear differential equation problems,

Most students in Turkish preferred integral multiplier method ($n = 225$; 75%) however, only 40% from the Ghanaian students preferred the integral multiplier method. In general, instead of finding the integral multiplier and transforming the equation to a complete equation and then solving, the majority of the two groups tend to solve it with $y = e^{-\int Pxdx} [\int e^{\int Px} q(x) dx + c]$ equation.

Question 14: Solve $y' + \frac{1}{x}y = \cos x$. $y = e^{-\int Pxdx} [\int e^{\int Px} q(x) dx + c]$ and $\vartheta(x) = e^{\int \frac{1dx}{x}} = e^{\ln x} = x \rightarrow y = x^{-1} [\int x \cos x dx \text{ and partial integral leads to } x^{-1} [x \sin x + \cos x + c]$.

2- Certain number of students from Turkey used Bernoulli $y = u(x) \cdot \vartheta(x)$ translation method ($n = 57$; 19%), however, in Ghanaian certain the majority of the students used Bernoulli $y = u(x) \cdot \vartheta(x)$ translation method ($n = 210$; 70%). This indicates that students in Ghana were familiar with this approach in class.

3- A few number of students from Turkey used Lagrange constant substitution method ($n = 13$; 4%) while those from Ghana the number of students were higher compared with the other group ($n = 30$; 10%). It could be suggested in both groups that the students preferred the methods used by their instructors instead of understanding the different methods. The authors in both countries observed that when they concentrated on certain methods after introducing all methods in their classes, those students neglected the methods that were not used by the instructor.

In problems that could be reduced to linear equations,

1- It was observed that the success rate was 70.6% with Bernoulli differential equation ($n = 212$) in the part of Turkey students while those from Ghana achieved 68.6% on the same concept. It is interesting to observe that the two groups most significant difficulty they experienced was the ($z = y^{1-n}$) translation.

Question 17: Solve $2x \left(xy^2 - \frac{1}{y} \right) y' = 1$ differential equation. When both sides are reduced $2x^2 + y^2 - \frac{1}{y} 2x = \frac{dx}{dy}$ by x^2 , from $x'^{x^{-2}} + \frac{2}{y} x^{-1} = 2y^2$, using the $z = x^{-1}$ translation, the solution $2xy^3 + cxy^2 = 1$ is obtained.

2- The two groups made translation by mistaking in questions where one par-

ticular solution is known in Riccati differential equations ($n = 198$; 66%) and ($n = 210$; 70%), Turkey and Ghana respectively.

Question 18: $y' - y^2 + \frac{1}{x}y + \frac{1}{x^2} = 0$.

2- In Riccati differential equations, those students from Turkey made mistakes in $y = y_1 + \frac{1}{u}$ variable translation ($n = 198$; 66%) while those students from Ghana ($n = 190$; 63.3%) made similar mistakes. Question 18: Solve the Riccati differential equation $y' - y^2 + \frac{1}{x}y + \frac{1}{x^2} = 0$ with a particular solution of $y = \frac{1}{x}$. When the $y' = -\frac{1}{x^2} - \frac{u'}{u^2}$ values are written in the equation and the equation is simplified, $u' + \frac{u}{x} = -1$ linear differential equation is obtained. With $U.x = \int -1.x dx$, $\frac{x^2}{yx-1} = -\frac{x^2}{2} + c$ general solution is obtained. ($n = 57$; 19%) for Turkey students and ($n = 54$; 18%) for Ghanaian students.

The success rate was 77.3% in Riccati questions with two, three and four known particular solutions using the obtained equations ($n = 232$) for Turkish students while those from Ghana ($n = 224$; 74.6%) achieve same success of those questions stated. However, in questions without any known particular solution, only 19.3% and 17.3% students from Turkey and Ghana respectively could translate into a second degree linear simple differential equation with $y' = \frac{u'}{pu}$ translation where $p \neq 0$ ($n = 58$). Question 19: When no particular solution is known, find the approximate solution for $y' - y^2 + 2xy - x^2 - 1 = 0$. When $y = -\frac{u'}{u}$ then $y' = \frac{u''u + u'^2}{u^2}$ expressions are written in the equation and the required simplifications are conducted, let us assume that in second order variable coefficient linear equation $u'' + 2xu' + u(x^2 + 1) = 0$, the solution is $u = \sum_{n=1}^{\infty} a_n x^n$, then when $u' = \sum_{n=1}^{\infty} n a_n x^{n-1}$ and $u'' = \sum_{n=1}^{\infty} n(n-1) a_n x^{n-2}$ derivatives are written in the equation, the a_n dependent particular solution $\sum_{n=1}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=1}^{\infty} 2n a_n x^n + \sum_{n=1}^{\infty} a_n (x^{n+2} + x^n) = 0$ is obtained.

The achievement rate was 88% and 86.6% respectively for Turkish and Ghanaian students respectively in first order high degree differential questions using the factorization method ($n = 264$) and ($n=260$). Question 20-A: Solve $y'^4 - (x + 2y + 1)y'^3 + (x + 2y + 2xy)y'^2 - 2xyy'$ equation. When $y' = p$ and polynomial factorization is conducted, then $p(p-1)(p-x)(p-2y)=0$. When the integral of each factor is taken, the $(y-c)(y-x-c)(y-\frac{x^2}{2}-c)(y-ce^{2x})$ general solution is obtained. The students from the two groups were challenged in differential questions that could be solved for the dependent variable ($n = 59$; 19.6%) and ($n = 60$; 20%) Turkey and Ghana respectively. Question 20.B: Find the general solution for $4y'^3 = 27(xy' - y)$ equation. Since $y' = p \rightarrow 4p^3 = 27(xp - y) \rightarrow y = -\frac{4p^3}{27} + xp$, it could be solved for y . When it is derived for x and required simplifications are conducted, then $\frac{dp}{dx}(x - \frac{4}{9}p^2) = 0$. When the integral of each

multiplier is taken separately, then $y = \pm x^{\frac{3}{2}} + c$ general solution is obtained. The achievement rate for Turkish students was quite low in Clairaut and Lagrange equations ($n = 67$; 22.3%) while the achievement rate for Ghanaian students were relatively high ($n=120$; 40%)

Question 20.C: Find general and unique solutions for $2y = 4xp + p^2$ differential equation if available. $2y = 4xp + p^2 \rightarrow 2 \frac{dy}{dx} = 4p + 4x \frac{dp}{dx} + 2p \frac{dp}{dx} \rightarrow -p = (p + 2x) \frac{dp}{dx}$ and $p+2x=$ and $p+2x \neq 0$ do not provide $y = -x^2$ equation, it is not a unique solution. When their conditions are analyzed separately, and when p is eliminated in $3xp^2 + p^3$ equation and the given $2y = 4xp + p^2$ differential equation, $8x^2 + 4y^2 - 12xy = c$ general solution is obtained. Those students from Ghana did not perform well because they failed to analyzed the conditions separately ($n=60$; 20%).

In differential application questions,

1- It could be suggested that the students from Turkish experienced difficulties in plotting the adequate figures in geometric applications ($n = 227$; 75.6%) while for Ghanaian situation the difficulties were relatively better ($n = 180$; 60%). Question 21.A: Find the curve family with a constant area below the normal. When the equation obtained with similarities is used; the solution is $s_n = |f(x) \cdot f(x)'| \rightarrow yy' = a$ and $y^2 = 2ax + c$ parabolic family.

2- The students were successful in increase/decrease questions ($n = 237$; 79%) for those from Turkey and similar in the case of those from Ghana ($n = 242$; 80.6%). It could be suggested that both groups comprehended $N(t)$ increasing or decreasing item count and the rate of change $\frac{dn}{dt} = kn$ k ratio constant equation . Question 21.B: TL 20000 was deposited to a savings account with 5% interest. Based on compound interest, what will be the account balance after 3 years? When the $\frac{dn}{dt} - 0.05n = 0$ equation is solved, for $n(t) = ce^{0.05t}$ and $t=3$, $N(t)=20\,000e^{0.05(3)}=23,236$.

3- It was observed that the two groups comprehended the Newton's concept with $\frac{dT}{dt} = -k(T - T_m)$ where T : object temperature, T_m : environment temperature, however they made calculation errors particularly those students from Ghana. Question 21.C: An object with an unknown temperature is put in a room with $30^\circ F$ constant temperature. If the temperature of the object is $0^\circ F$ after 10 minutes and $15^\circ F$ after 20 minutes, what was the unknown baseline temperature?

$\frac{dT}{dt} + kT = 30k \rightarrow T = ce^{-kt} + 30$ for $t=10$, $ce^{10k} = -30$ for $t=20$, $15=ce^{-20k} + 30 \rightarrow k = \frac{1}{10} \ln 2 = 0.069$ and $c=-60$, then $T_0 = -60e^{(-0.069) \cdot 0} + 30 = -30^\circ$.

4- The two groups of students solved free fall problems using the second Newton law that "the acceleration of an object as produced by a net force is directly propor-

tional to the magnitude of the net force, in the same direction as the net force" (n = 229; 76.3%) and (n = 270; 90%) Turkey and Ghana in that order.

Question 21.D: An object with 5 kg mass falls from 100 ft with a speed of zero. When air resistance is neglected, a) find the position and speed equation for any given time t, and b) find the time it takes to reach the ground. a) $\frac{dv}{dt} = g$, $v = gt + c$ $t=0$, if $c=0$ and $v=32t$ then, using $v=\frac{dx}{dt}$, the position equation is $x=16t^2+c_1$, $t=0$ için $x=16t^2$ b) When $x=100$ then $t=?$ $X=16t^2 \rightarrow t = 2,5 \text{ sn}$

5- Dilution and electrical problems are a differential movement, however these require chemical and electrical/electronic knowledge. It could be stated that interdisciplinary collaboration could improve achievement in these problems (n = 167; 55.6%) and (n = 198; 66%), Turkey and Ghana respectively.

Question 21. E: There is a 100 gal solution that included 20 lb salt in a tank. At time $t=0$ pure water is poured into the tank with a speed of 5gal/min, and at the same time well mixed solution is rapidly discharged from the tank. Find the salt content in the tank at any given time t. When the equation obtained in the class is used; Since, $v_0=100$ $a=1$, $b=1$ and $e=f=3$, $\frac{dQ}{dt} + \frac{f}{v_0+(e-f)t} Q = be \frac{dQ}{dt} + 0.03Q = 3$. When this equation is solved, when $Q = ce^{-0,03t}+100$ and $t=0$ then $c=-99$, therefore $Q=-99e^{-0,03t}$. Since $Q=2$, then $2=-99e^{-0,03t}+100$ $t=0,338$

In second order linear equation problems with constant coefficient, the students from both groups did not experience problems since these formulas were comprehended (n = 237; 79%) and (n = 244; 81.3%), Turkey and Ghana in that order. Naturally, calculation errors persevered for the two groups. Question 22.A: Solve $\frac{d^2y}{dx^2} + 18y = 0$. $m^2+18=0$ $m=4i$ then $y=A\cos 4x+B\sin 4x$. Question 22.B: Find the general solution for $\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 4e^{5x}$ (n = 198; 66%). $m^2 + 14m + 49 = 0 \rightarrow m_1 = m_2 = -7$ Since the integral function is $y=e^{-7}(A+Bx)$, $f(x)=4e^{5x}$, the special integral is $y=ce^{5x}$. When it is written in the equation, then $c=1/36$, thus the special integral is $y=1/36 e^{5x}$ and general solution is $y=e^{-7x}(A+Bx)+\frac{1}{36}e^{5x}$. Here, problems were experienced in finding the equation based on the position of the special integral for both groups and it was serious in the case of Ghanaian students (n = 60; 20%).

When it is accepted that first order and high degree differential equations in the form of $f(x,y,y')=0$ could not be solved for y generally, the two groups of students did not experience any difficulties in questions related to singular n solutions since the concept and the operation was comprehended and the calculations were relatively shorter (n = 234; 78%) and (n = 270; 90%) for Turkish and Ghanaian students in that order. Question 22: Find the singular solution for the differential equation $y = xy' -$

$\sin y'$.

P is eliminated in $y' = p$, $f(x, y, p) = 0$ and $\frac{\partial f(x, y, p)}{\partial p} = 0$ and $y - xp + \sin p = 0$ and $-x + \cos p = 0$ equations to obtain $y = x \arccos x - \sqrt{1 - x^2}$.

$F(x, y, c) = 0$ describes a family of curves. In this family of curves, different curves appear on the plane as c parameter changes. If a curve Z that is a tangent to all these curves is present, it is called the envelope of F family of curves. This envelope is also a singular solution. However, each singular solution is not an envelope. 174 students from Turkey solved these type of problems (58%) while 168 students (56%) solved same problem from Ghana. Question 23: Find the equation for the singular solutions of the curve $y = (x - c)^2 + 3$ and determine whether the singular solution is an envelope. c is eliminated using the equation $y - (x - c)^2 - 3 = 0$ and its derivative $2(x - c) = 0$ and $c = 3$ $y'^2 = 4(y - 3)$ differential equation is obtained and since it provides $y = 3$, it is a singular solution.

Another equation, which is more general when compared to the Clairaut equation that could be solved for y , is the Lagrange equation: $y = xg(y') + f(y')$. In Lagrange equations the students from Turkey achieved 185 (61.6%) however, in Ghana 200 students (66.6%) achieved similar rate. Question 24: Find the general solution for the differential equation $y = -xy' + y'^2$. For $Y' = p$, if $\frac{dy}{dx} = -x \frac{dp}{dx} + 2p \frac{dp}{dx}$, then $2p = (-x + 2p) \frac{dp}{dx}$ where if $2p = 0$ then $y = 0$ is the singular solution. For $2p \neq 0$, if $\frac{dx}{dp} + \frac{1}{2p}x = 1$ then $x = \frac{2p}{3} + \frac{c}{\sqrt{p}}$ and $y = \frac{p^2}{3} - c\sqrt{p}$ parametric equation is obtained.

As is known, in equidimensional equations, each of the $y, y', y'', \dots$ expressions could be matched with the same number m , the number m is called the dimension of these expressions. When each term in the equation is in the same dimension, the equation is described as an equidimensional equation. The two groups of students' achievement rate were 63% ($n = 189$) were 55.6% ($n = 167$) for Turkey and Ghana in that manner. In the equidimensional equation questions, despite both groups experienced difficulties in translation operations. Question 25: Find the general solution for $yy'' - y'^2 = 6xy^2$ differential equation. Since y, y', y'' was equidimensional, with $y = e^u$ translation $y = ce^{x^2 + c_1x}$ general solution is obtained.

The achievement rate respectively for Turkey and Ghana were 44% and 48.3% in questions related to the Sarrus method in complete differential equations ($n = 132$) and ($n = 145$). As is known, if the n th order differential equation $f(x, y, y', \dots, y^{(n)}) = 0$ could be written to equal the derivative of the $(n-1)$ th order $g(x, y, y', \dots, y^{(n-1)}) = 0$ equation, then f is described as a complete differential equation. Sarrus method is used to determine whether the equation is complete. Question 26: Find the general solution for differential equation $(1 + x^2)y'' + 2xy' - 2x = 0$. The equation $f = (1 + x^2)y'' +$

$2xy' - 2x = 0$ is linear based on its highest derivative and when one step smaller of the term that includes that derivative $u_{1=(1+x^2)y'}$ is derived and simplified and the integral of $dy = \frac{c_1+x^2}{1+x^2} dx$ is taken, the $y = (c_1 - 1)\arctan x + x + c_2$ general solution is found.

The equations that include the derivative of more than one dependent variable based on an independent variable are described as ordinary differential equations. It could be argued that the students from both countries did not experience difficulties in differential equation system problems ($n = 258$; 86%) and ($n = 263$; 87.6%) respectively. Question 27: Solve $\frac{dx}{dt} = -2x + y$ and $\frac{dy}{dt} = -3x + 2y + 2\sin t$ equation system. (çözüm eklenecek)

Laplace transformation is an integral transformation and an important method used in applied sciences. In solution of a differential equation given with baseline conditions (which is sometimes quite difficult), the equation is initially transformed into an algebraic equation using Laplace transformation and after the algebraic equation is solved, the product is transformed back into a differential equation with reverse-Laplace transformation to obtain the solution. In solution of differential equations with Laplace transformation, the students in both countries did not experience difficulties except for calculation errors ($n = 279$; 93%) and ($n = 267$; 89%). Perhaps students in both countries apply this concept in other disciplines and therefore very familiar with Laplace transforms. Question 28- A: Find the Laplace transformation below: $F(t) = \begin{cases} -1; & 0 \leq t \leq 2 \\ 3; & t \geq 2 \end{cases}$ $L[f(t)] = \int_0^\infty f(t)e^{-st} dt$ and $|f(t)| = \int_0^2 (-1)e^{-st} dt + \int_2^\infty 3e^{-st} dt = \frac{4e^{-2s}}{s}$. In questions that included Laplace transformation in differential equations with both constant and variable coefficients, the groups experienced problems due to operational knowledge ($n = 149$; 49.6%) and ($n = 153$; 51%), Turkey and Ghana respectively. Question 28-B: Solve the $y'' + 6y' + 5y = 0$; $y(0) = 0$, $y'(0) = 1$ baseline value problem. When the Laplace transformation of the equation is taken; if $L[y'' + 6y' + 5y] = L[0]$ then $s^2L[y] - sy(0) - y'(0) + 6(sL[y] - y(0)) + 5L[y] = 0$, with $y(0)$ and $y'(0) = 1$ baseline conditions, $(s^2 + 6s + 5)L[y] = 1$, $L[y] = \frac{1}{s^2 + 6s + 5} = \frac{1}{4} \left(\frac{1}{s+1} - \frac{1}{s+5} \right)$ with reverse-Laplace $\frac{1}{4}e^{-t} - \frac{1}{4}e^{-5t}$ is found.

4. Discussion and Conclusion

The findings obtained with the classification of differential equations course exam sheets, assignments and projects submitted by pre-service teachers and engineers in the last five years based on the questions demonstrated that 71% of the students selected x as the independent variable and y as the dependent variable and experienced difficulties in problems when these functions were reversed (Question 17). Similarly, in Ghana same questions 74% students selected x independent and y as the dependent

variable and they encountered difficulties in reversed form as in (Question 17). In both cases this difficulty could be reduced by the instructors, by focusing on different examples and providing opportunities for students to practice adequately. In determination of the order and degree in differential equations, in equations with a fractional exponent, 45.3% and 40% from Turkey and Ghana respectively determined the order and degree of the differential equation incorrectly or stated that it had no order and degree. The polynomial property of the function should be emphasized in determination of the degree and order (Question 2). About sixty(60) percent of Turkish students experienced difficulties in determining whether the differential equation was linear whilst those students from Ghana was Fifty-eight (58) percent had same difficulties for showing a differential equation is a linear or not . Both group ignore the rule that in order to be linear, the degree of the derivatives should also be one and pay attention only to the multiplication of the dependent variable and its derivatives (Question3). The linearity property is an important property to solve several differential problems. It should be reinforced with abundant examples and frequent practices.

These students from the two groups preferred general solutions and assigning arbitrary values to the constants in general solutions in the solution of differential equations. It could be stated that these students ignored the solutions without constants (singular) (Question 4). As the solution of the differential equations can be represented by a family of curves, 65% of the pre-service engineers and teachers in Turkish, who experienced difficulties in plotting the graphs, failed while in Ghana 62% of the pre-service teachers and engineers encountered the same problem. Improvement of these groups knowledge on analytical geometry, especially in the faculty of engineering and faculty of technical education respectively, could improve achievements (Question5). Sixty-eight percent (68%) were successful in differential equations that could be solved directly with integral except calculation errors, 32% of the students from Turkey experienced difficulties in problems that could be separated into variables. In Ghana 80% of the students were also successful solving differential equations that required direct integration approach with minimal calculation errors, 20% of the Ghanaian students encountered some challenges with problems required separating of variables. In order to improve the achievement in problems that require separation into proper fractions, abundant examples should be given for algebraic operations (Questions 6-7).

Since most differential equations require derivation and integration, the failure rate were 47% and 40% respectively for Turkey and Ghana (Questions 8, 9). Formulas should be given with proofs and requirement of the formula proofs in the exams should raise the focus of the groups of students on these difficulties. It was observed that the achievement relatively increased when the formulas that could lead to confu-

sions were printed on the exam sheets in certain exams. This topic can be discussed in academic circles or selected as a study topic. In homogenous functions with the same degree for Turkish students, the success in the solution of differential equations with $u = ux$ conversion was 67.3% when $\Delta = 0$, while it decreased to 35.6% when $\Delta \neq 0$. For Ghanaian students on the same concept, the success on homogenous function with $u = ux$ conversion was 73.6% when $\Delta = 0$, while it reduced to 33.6% when $\Delta \neq 0$. Here, it was not understood that axis translation includes nullification of the constants and to obtain an equation with $\Delta = 0$ and to obtain the real solution with reverse translation (due to the lack of graph plotting knowledge) (Question 10). It would be beneficial to instruct this topic by plotting lines on the analytical plane. It could be stated that they preferred the second equation that provides the shortest solution among the equations obtained during the solution of complete differential equations and their disposition was towards computational learning rather than conceptual learning, which led to an increase in computational errors and a decrease in achievement. Students should be encouraged to use the concept-oriented first equation. To help the solution of non-complete differential equations, the rate of those who experienced difficulties in determining whether it was dependent on x or y was 59.3% and (60%) respectively for Turkish and Ghanaian students (Question 11). Plenty of examples should be provided for pre-service engineers and teachers to allow them to select the accurate one by trial.

Both groups did not perform well in functions that require expansion to power series and only 27% of the Turkish students could do this problem and in Ghana the situation was not different as 24% of the students were able to solve that problem (Question 12). This suggests that the teaching of calculus should be strengthened in both institutions by focusing on students –based learning approach which will compel students to actively involved in learning process. Tutorials should be organized by the respective institutions.

The 75% of the Turkish students were familiar with the multiplier method while their counterpart from Ghana only 40% were conversant with this approach (Question 13). The students should be presented with several approaches and be given opportunity to practice and followed through series with tutorial.

In Turkey only 19% of the students used Bernoulli $y = u(x) \cdot \vartheta(x)$ translation method while those who took part in the course from Ghana the majority used same approach (70%) (Question 14). This means that instructors have the duties to expose students to variety of approaches that are used to solve differential equations. In addition, a lot of exercises and prompt feedbacks must be given to the students in order to improve their learning. Frequently, instructors must be retrained so that they will be to understand new trend of education where most teaching and learning are taking place online. It was established that only 4% utilized Lagrange constant substitution method

while in Ghana only 10% were familiar with the method. This indicates those possibly these students were not taught and that explain why instructors must ensure that students are provided all the necessary information needed in the teaching and learning of ODES.

In Riccati differential equation problems, the success rate was 66% for students from Turkey while those students from Ghana was 63.3% despite the errors in the transformation phase in the solution of an equation with a known particular solution. The status could be improved by solving several examples (Question 18). In equations with two, three, and four known particular solutions, the achievement rate for students in Turkey was 73.3% and that was in Ghana 74.6% when obtained equations were used, however the success rate declined to 19.3% and 17.3% for Turkey and Ghana students respectively in Riccati differential equations without known particular solution. The achievement declined due to $y' = \frac{u'}{pu}$ transformation for $p \neq 0$ (Question 19). Approximation methods should be emphasized.

The students from Turkey achieved the 88% success rate and those students from Ghana achieved 86.6% in first order high degree differential equation problems with the implementation of factorization (Question 20.A). Their achievement rate declined to 19.6% and 20% respectively for Turkey and Ghana when they solved the problem with the dependent variable due to calculation errors (Question 20.B). The achievement rate in Clairaut and Lagrange questions was 22.3% and 40% for Turkish and Ghanaian students respectively due to the difficulties in analyzing the problem stages separately (Question 20.C).

In differential application questions,

1- In geometric applications, the achievement rate was 59.3% for Turkish and 60% for Ghanaian students due to errors in plotting geometric shapes, however the achievement rate increased to 75.6% and 60% for Turkish and Ghanaian students respectively in the calculation of tangent, subtangent, normal and subnormal distances of a curve using form and similarity correlations although it is a geometric application (Question 21, B).

2- In increase and decrease, free fall and temperature questions, 79% and 80.6% for Turkish and Ghanaian students respectively of pre-service teachers and engineers comprehended the topics better due to physical applications (Questions 21. C-D-E). The students stated that they comprehended the reasons for physics equations and how these equations were obtained for first time with differential applications.

3- In dilution and electrical questions, the achievement rate was 55.6% and 66% respectively for Turkish and Ghanaian students due to the requirement of chemistry and electricity knowledge. Thus, it was observed that collaboration with field instruc-

tors could improve the achievements (Question 21, F).

It could be argued that the students were successful (78%) and 90% for Turkish and Ghanaian students respectively in first-order high degree differential equation questions due to the comprehension of the concept and the operations (Question 22). It could be concluded that they experienced difficulties in plotting accurate forms in singular solution and envelope questions (58%) and (56%) for Turkish and Ghanaian students in that order (Question 23), and their knowledge on plotting the functions and analytical geometry should be enforced. In Lagrange problems that could be solved for y and are a more general form of Clairaut equation, the achievement rate were (61.6%) and (66.6%) respectively for Turkish and Ghanaian in which they experienced difficulties in the two stages ($2p = 0$ and $2p \neq 0$) and to obtain the parametric equation (Question 24). In an application, it was determined that this could be corrected by solving several examples. A 63% and 55.6% for both groups Turkey and Ghana respectively achievement rate was obtained in equidimensional differential equation problems. Pre-service teachers and engineers usually made mistakes in the $y = e^u$ transformation. This was observed in all questions that required a transformation and was due to lack of calculus knowledge (Question 25). The achievement rate was 44% and 48.3% for Turkish and Ghanaian students in complete differential equation questions in Sarrus method due to calculation errors (Question 26). In equation systems, 86% and 87.6% for students from Turkey and Ghana respectively and obtained the solution with a solution similar to that of the first order equation systems with two unknowns. It could be argued that they were successful by applying previous knowledge on equation systems to the new solution method (Question 27).

It is not always easy to find the general solution for a differential equation given with baseline conditions. As an alternative method, it could easily be solved by transforming the equation directly into an algebraic equation by Laplace transformation and to obtain the solution of the differential equation by the inverse-Laplace transformation. In these types of questions, it was determined that students did not experience difficulties in Laplace transformation and operation knowledge, however the success rate declined in reverse-Laplace transformation due to operation knowledge (Question 28). When pre-service teachers and engineers encountered this alternative method for the first time, it may be beneficial to instruct these topics in more detail. In the present study that analyzed the difficulties encountered by pre-service teachers and engineers during the last five years in the differential equations course, mainly general difficulties were investigated, and a limited comparison was conducted based on the faculty and department. The findings obtained with the analysis of the exam questions demonstrated that the students from Turkey achievement was mediocre in the field of differential equations, which is an applied course that require basic mathematics

knowledge, the faculty of education students were more successful when compared to engineering students by 76% due to the higher score requirement of the education faculty when compared to the faculty of engineering in the university admissions exam, the same difference favored primary school mathematics teaching department by 67% in the education faculty and the civil engineering department by 79% in the engineering faculty based on the exam sheet findings.

In Ghana the same study was conducted in faculty of technical education where from the average scores obtained from their examination scripts, the construction technology education was more successful by 74% followed by 71% Auto-Mechanical engineering education and also 70.8% was electrical/electronic engineering education and finally Wood technology education 69.2%. The majority of the students admitted into this faculty do not come from the normal conversion secondary schools where they offer elective mathematic but rather from the Ghanaian Technical School where the mathematics they study is very weak.

Analysis based on the years scrutinized in the study reflected that interdisciplinary collaboration of the instructors of the same course for five years led to an increase in achievements and the opposite conditions led to a decrease in achievements.

General Recommendations

The mathematics achievement rates should be considered in the placement sections of the students who would be admitted in the departments of mathematics and especially in the engineering faculty departments that include advanced applied mathematics courses. For the case of Ghana, the general requirements into faculty of technical education must include elective mathematics at the senior level. One of the most important factors in the development of a nation is science and technology education and the cornerstone of this education is mathematics instruction. The instructors should be selected carefully in these departments and their knowledge should be improved with training in the country and abroad in both countries. It was considered that it would be beneficial to take compulsive mathematics courses for the achievement of the students that would be admitted from two-year academics to engineering faculties via vertical placement. These students create a problem for the learning balance in the classes due to lack of knowledge in Turkey. Since the students mainly focus on the threshold, these courses should be considered as threshold courses for all students in Turkey. For the case in Ghana, students admitted into second year of the faculty programmes who already have diploma certificate should be made to take again all the calculus courses alongside with differential equations course. The knowledge that mathematics is used in certain basic events enables students to develop a positive attitude towards mathematics in both groups. Thus, this course should be instructed at least for two semesters in mathematics teaching programs in education faculties and

should be instructed at a basic level in high schools, especially in science high schools in both countries as against only one semester.

Recommendations for Future Studies

The scope of the present study was only ordinary differential equations. Future studies could be conducted on partial differential equations and for Ghana expand beyond one faculty. Future studies both countries on in depth investigation of certain special topics could be conducted since differential equations is an extensive field; i.e., future studies on errors in differential applications or Laplace applications could be conducted and studies on solution proposals could contribute to the literature.

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